Geometric invariance for synthetic aperture radar (SAR) sensors

Vincent J. Velten

Air Force Research Laboratory, AFRL/SNAT, 2010 Fifth St., WPAFB, OH 45433-7001

Approved for Public Release; distribution unlimited.

ABSTRACT

Synthetic Aperture Radar (SAR) sensors have many advantages over electro-optical sensors (EO) for target recognition applications, such as range-independent resolution and superior poor weather performance. However, the relative unavailability of SAR data to the basic research community has retarded analysis of the fundamental invariant properties of SAR sensors relative to the extensive invariant literature for EO, and in particular photographic sensors. This paper develops the basic geometric imaging transformation associated with SAR from first principles, and then gives an existence proof for several geometric scatter configurations which give rise to SAR image invariants.

Keywords: synthetic aperture radar, scattering centers, geometric invariance, model-based vision

1. INTRODUCTION

Current research into automatic target recognition using SAR has tended to focus on the photometric as opposed to geometric properties of SAR sensors. Template based approaches wherein recognition is obtained by matching the unknown target image chip to a precomputed reference image are popular,[1,2] but suffer from a lack of scaleability when a large number of target classes or extensive variations in target appearance are encountered. This is a fundamental limitation of such view-centered methods.[3]

An alternative to view based approaches to automatic target recognition (ATR) is an object-centered approach. In the object-centered approach, recognition is accomplished by comparing sensed data to stored three dimensional (3D) models that ideally capture all of the essential signature variability of the target as imaged by a given sensor.[4] An example of such an approach for SAR ATR is embodied in the moving and stationary target acquisition and recognition or MSTAR system.[5] In MSTAR, predicted peak and region features are matched to SAR image extracted peaks[6] and regions in an iterative fashion to allow for variation in the sensed object's signature. To avoid the computational cost incurred by attempting on-line calculation of radar scattering, MSTAR utilizes a data compression technique[7] to extract 3D scattering centers from predicted xpatch-t synthetic SAR data. Although the MSTAR approach has been successful in overcoming some of the scaleability limitations of the template approach, it does not exploit all of the information inherent in the explicit consideration of 3D models and their projection into a two dimensional (2D) image.

There has been considerable interest in the object-centered approach to ATR for optical sensors, and in particular for photographic sensors. Numerous examples can be found in the recent image understanding literature.[9,10,11,12] An object-centered technique of particular interest is geometric invariance.[13] Simply stated, a geometric invariant is an image measurable property of a 3D configuration that is unaffected by imaging viewpoint. Since it's introduction to computer vision in 1988,[14] geometric invariance has been extensively developed for vision applications.[15] Geometric invariants can be derived for single sensor views of a 3D object or as a relationship across multiple views. One can term these two types of invariant monoscopic and stereoscopic invariants respectively. For any given imaging sensor, there are generally a richer array of stereoscopic invariants available than monoscopic invariants, but this numerical advantage is offset by the need for image correspondence in order to use stereoscopic invariants. Geometric invariants can be absolute, or completely independent of the particular imaging geometry (i.e., independent of sensor position and orientation relative to the object), or they can be relative to a particular imaging geometry. Relative invariants are typically of mathematical interest and are not generally useful for object recognition applications, except to the extent that they can be used in combination to generate absolute invariants. Geometric invariants can be powerful features for object recognition since they alleviate any need to search for a match over object pose. To date, little of geometric invariant theory has been applied to radar frequency (RF) sensors. Payton and Barrett have examined invariant structures in stereo HRR sensing,[16] and Binford has examined persistent scattering of BTR70 vehicle road wheels in xpatch-t data as an invariant indexing mechanism.[17] Stuff is exploring invariant constraints for application to 3D SAR sensor signal processing, but little of his work has appeared in the literature as yet.[18]

One potential extension of the MSTAR approach is to consider the *geometric* effects of the SAR sensor transformation in conjunction with the 3D scattering center concept. By addressing the algebraic nature of the SAR imaging transformation it becomes possible to determine configurations of 3D scattering points that give rise to monoscopic SAR geometric invariants.

The development of the basic mathematics of monoscopic SAR geometric invariants is the primary thrust of this paper.

2. A HIGH RANGE RESOLUTION (HRR) RADAR INVARIANT

Before developing SAR geometric invariants, it is useful to first consider the one dimensional (1D) radar case. This allows for a simpler analytic development, and the result naturally extends to the 2D SAR case. This also happens to be the historical order in which the research progressed. Figure 1 gives the basic imaging geometry for the HRR sensor.
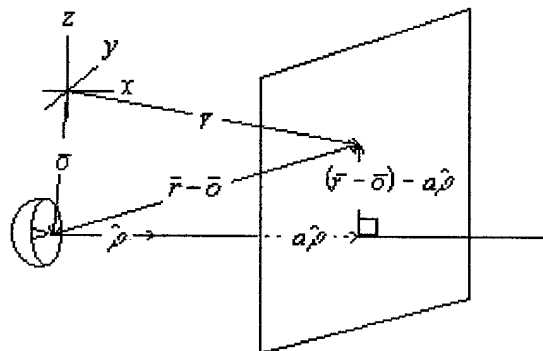

Figure 1. HRR geometric transformation

In this paper, we shall assume that any time mention is made of a '3D point' we are referring to a scatterer whose phase center is located at that point in 3D space, and not just any arbitrary 3D location devoid of content. That is to say, we are focusing our attention on sources of RF scattering that can be placed at a particular 3D location.

We approximate the HRR sensor transformation of a point in 3D space as the perpendicular projection of that point onto a vector along the radar look direction. Note that all points in any plane perpendicular to the look direction are projected into the same point in the HRR return. This is an approximation because the true transformation projects all points of equal distance from the radar (within the radar main beam) onto the radar look direction, so that the plane in our approximation should actually be a spherical surface. Our approximation is therefore equivalent to making a plane wave assumption for the radar wave front at the scale of typical target sizes. In a typical HRR return, the signal is divided into many discrete resolution cells that represent the coherent addition of all scatters within a range bin (in effect all scatterers in a 3D planar slab are projected and summed coherently into a single HRR range cell). We assume that the RF scattering is such that for at least some cells a single scattering point in 3D dominates it's corresponding HRR return cell.

Figure 2. A typical HRR return of an aircraft object

A typical HRR return is shown in figure 2. Since HRR is a one dimensional signal, it is apparent that monoscopic invariants can only be obtained as some function of the distances between HRR return peaks. Since invariants are inherently dimensionless, it is reasonable to consider ratios of HRR inter peak distances as a likely invariant structure. Our strategy is to determine if there exists a 3D arrangement of scatterers that would give rise to an invariant distance ratio in the corresponding 1D HRR projection of the scattering centers. From figure 1, the projection a of a 3D point $\bar{r}$ given in (x,y,z) coordinates into the 1D HRR signature is easily found given the radar look direction unit vector $\hat{p}$ by

$$\left[\left(\bar{r}-\bar{o}\right)-a\hat{p}\right]\cdot\hat{p}=0 \rightarrow a=\left(\bar{r}-\bar{o}\right)\cdot\hat{p} \quad (1)$$

where $\bar{o}$ is the arbitrary origin of the HRR coordinate system. Note that without loss of generality a can be assumed to include any scale factors needed for the HRR down range variable. This gives the fundamental HRR transformation projection constraint

$$a=f_{\hat{p},\bar{o}}\left(\bar{r}\right)=\left(\bar{r}-\bar{o}\right)\cdot\hat{p} \quad .(2)$$

We are interested in determining conditions under which we have distance ratio invariants in HRR. This generally implies at least three distinct points in 3D space. We denote distinct 3D points by $\bar{p}_i$, 3D Euclidean distances between any pair of two distinct points $(\bar{p}_i, \bar{p}_j)$ by $d_3(\bar{p}_i, \bar{p}_j)$, and the corresponding 1D HRR distances by $d_1(a_i, a_j) = |a_i - a_j|$. We can then relate the 3D and 1D distances between distinct points by

$$a_i - a_j = f_{\bar{p}_i}(\bar{p}_i) - f_{\bar{p}_i}(\bar{p}_j) = \left(\left(\bar{p}_i - \bar{o}\right) - \left(\bar{p}_j - \bar{o}\right)\right) \cdot \hat{\rho} = \left(\bar{p}_i - \bar{p}_j\right) \cdot \hat{\rho} = d_3(\bar{p}_i, \bar{p}_j)\hat{p}_{ji} \cdot \hat{\rho} \quad (3)$$

where $\hat{p}_{ji}$ is the unit direction vector from $\bar{p}_j$ to $\bar{p}_i$. Therefore

$$d_1(a_i, a_j) = |a_i - a_j| = \left|d_3(\bar{p}_i, \bar{p}_j)\hat{p}_{ji} \cdot \hat{\rho}\right| = d_3(\bar{p}_i, \bar{p}_j)\left|\hat{p}_{ji} \cdot \hat{\rho}\right| \quad (4)$$

Note that an added benefit of considering inter peak distances in the HRR signature is the elimination of the need to consider the HRR coordinate origin, so that we immediately gain invariance to 3D translation. A ratio of distances between dominant scatterers on the HRR line is related to the corresponding 3D distances between the scatterers by

$$\frac{d_1(a_i, a_j)}{d_1(a_k, a_l)} = \frac{d_3(\bar{p}_i, \bar{p}_j)}{d_3(\bar{p}_k, \bar{p}_l)}\frac{\left|\hat{p}_{ji} \cdot \hat{\rho}\right|}{\left|\hat{p}_{lk} \cdot \hat{\rho}\right|} \quad (5)$$

Therefore HRR distance ratios give an invariant if

$$\frac{\left|\hat{p}_{ji} \cdot \hat{\rho}\right|}{\left|\hat{p}_{lk} \cdot \hat{\rho}\right|} = \text{constant} \quad (6)$$

regardless of the orientation $\hat{\rho}$ of the radar look direction. Clearly a sufficient condition for this to arise is for $\hat{p}_{ji}$ and $\hat{p}_{lk}$ to be parallel vectors, that is, $\hat{p}_{ji} = \pm\hat{p}_{lk}$, and in this case the constant is unity and we have an absolute invariant. For arbitrary $\hat{\rho}$ it is readily seen that this must also be a necessary condition as well. This can be shown as follows. For ease of notation denote $\hat{p}_{ji}$ and $\hat{p}_{lk}$ by $\hat{p}$ and $\hat{q}$ respectively. Our constraint then is

$$\frac{\left|\hat{p} \cdot \hat{\rho}\right|}{\left|\hat{q} \cdot \hat{\rho}\right|} = f(\hat{p}, \hat{q}, \hat{\rho}) = \text{constant} \quad , (7)$$

and we are interested in the relationships between $\hat{p}$ and $\hat{q}$ satisfying this constraint for arbitrary $\hat{\rho}$, subject to $\hat{\rho} \perp \hat{q}$ and $\hat{\rho} \perp \hat{p}$ so that this constraint does not become undefined or trivially satisfied respectively. Let $\hat{\rho} = \hat{q}$. Then since $|\hat{p} \cdot \hat{q}| \le 1$, we have $f(\hat{p}, \hat{q}, \hat{q}) \le 1$. Similarly for $\hat{\rho} = \hat{p}$ we obtain $f(\hat{p}, \hat{q}, \hat{p}) \ge 1$. Therefore for arbitrary $\hat{\rho}$, $f(\hat{p}, \hat{q}, \hat{\rho}) = 1$. In this case

$$\left|\hat{p} \cdot \hat{\rho}\right| = \left|\hat{q} \cdot \hat{\rho}\right| \Rightarrow \left(\hat{p} \cdot \hat{\rho}\right)^2 - \left(\hat{q} \cdot \hat{\rho}\right)^2 = 0 \Rightarrow \left(\hat{p} \cdot \hat{\rho} - \hat{q} \cdot \hat{\rho}\right)\left(\hat{p} \cdot \hat{\rho} + \hat{q} \cdot \hat{\rho}\right) = 0 \Rightarrow \hat{p} \cdot \hat{\rho} = \pm\hat{q} \cdot \hat{\rho} \quad (8)$$

Since this equation must hold for arbitrary $\hat{\rho}$, it holds in particular for $\hat{\rho} = \hat{q}$ and so we obtain $\hat{p} \cdot \hat{q} = \pm 1$ or $\hat{p} = \pm\hat{q}$, i.e., parallel vectors. This proves that parallel vectors is both a necessary and sufficient condition for an absolute ratio distance invariant in HRR. Note that Payton and Barrett[16] derive a similar result for the case of three collinear points as a special case of a two view stereoscopic HRR invariant for four coplanar scattering centers, which they term a "Velten Invariant." As the name might imply, this invariant was first suggested by the author, and Payton and Barrett examined HRR invariants at the author's request under contract to AFRL.

3. SAR GEOMETRIC INVARIANTS

We begin as we did for the HRR case by deriving an approximation for the SAR 3D to 2D imaging transformation. As in the HRR case, this is an approximation of the SAR imaging projection, as the true physical situation the projection is also along a spherical front centered at the synthetic aperture phase center. However, for objects of limited extent such as vehicles, the

error introduced by assuming a perpendicular projection into the slant plane is negligible,[19] and we are in effect assuming that the point is in the far field of the synthetic aperture.
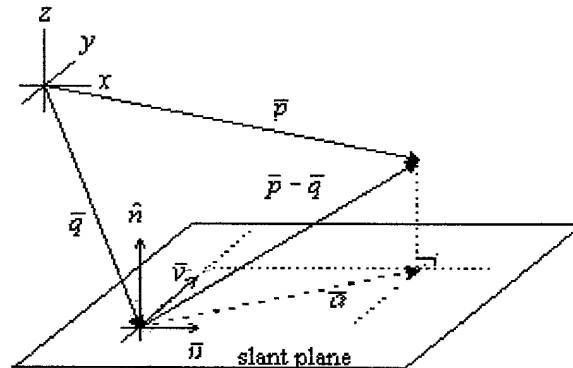

Figure 3. SAR imaging geometry

Denote 3D world coordinates as (x,y,z), general (column) vectors in 3D world coordinates by $\bar{r}$, and unit vectors in world coordinates by $\hat{r}$, where r is any Roman symbol. The SAR slant plane position and orientation are given by unit normal vector $\hat{n}$ located at $\bar{q}$ in the 3D world frame. We specify a 2D coordinate system lying in the slant plane (i.e., the SAR image) by perpendicular vectors $\bar{u} = s_u\hat{u}$ and $\bar{v} = s_v\hat{v}$, where s_u and s_v are slant plane coordinate scale factors and $\hat{u}$ and $\hat{v}$ are unit vectors. Finally, we denote locations in the 2D SAR image coordinate system using Greek symbols e.g. $\bar{\alpha}$, where $\bar{\alpha}$ is an ordered pair specifying components in the $(\bar{u},\bar{v})$ directions. As in the HRR case, we will assume that SAR image peaks are generally the result of a dominant 3D scatterer.

Approximate the SAR image of a point at $\bar{p}$ as the perpendicular projection of $\bar{p}$ into the slant plane. We wish to determine the SAR image coordinates of $\bar{p}$, or $\bar{\alpha}$ in figure 3, as a function of the know slant plane geometry. Clearly $\bar{\alpha}$ is the slant plane perpendicular image of $\bar{p} - \bar{q}$. Since $\bar{a} \equiv \bar{p} - \bar{q}$ is a vector whose origin is at the slant plane origin, we can proceed by finding the change of coordinate matrix T than transforms vectors in (x,y,z) coordinates with their origin at $\bar{q}$ into (u,v,n) coordinates. Clearly $T = \begin{bmatrix} \bar{u} & \bar{v} & \hat{n} \end{bmatrix}^r$, so that $\bar{a}$ transforms to $\bar{a}' = T\bar{a} = (\bar{u}\cdot\bar{a}, \bar{v}\cdot\bar{a}, \hat{n}\cdot\bar{a})^r$ in the (u,v,n) system. We can then easily determine $\bar{\alpha}$ from

$$\bar{\alpha} = P\bar{a}' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\bar{a}' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}T\bar{a} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} u_x & u_y & u_z \\ v_x & v_y & v_z \\ n_x & n_y & n_z \end{bmatrix}\bar{a} = \begin{bmatrix} u_x & u_y & u_z \\ v_x & v_y & v_z \end{bmatrix}\bar{a} = (\bar{u}\cdot\bar{a}, \bar{v}\cdot\bar{a})$$

. (9)

Note that we can express T as the product of scaling and rotation matrices T_s and T_{rot} respectively.

$$T = T_s T_{rot} = \begin{bmatrix} s_u & 0 & 0 \\ 0 & s_v & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \hat{u}^r \\ \hat{v}^r \\ \hat{n}^r \end{bmatrix}$$ (10)

Therefore the SAR image coordinates $\bar{\alpha}$ of a 3D point at $\bar{p}$ are

$$\bar{\alpha} = \begin{bmatrix} s_u & 0 \\ 0 & s_v \end{bmatrix}\begin{bmatrix} \hat{u}^r \\ \hat{v}^r \end{bmatrix}(\bar{p} - \bar{q})$$

. (11)

For the case of a SAR system with 'square' pixels, i.e., for which the along and cross range image dimensions are of the same resolution, we have $s_u = s_v \equiv s$ and the SAR image coordinate expression becomes

$$\bar{\alpha} = s \begin{bmatrix} \hat{u}^r \\ \hat{v}^r \end{bmatrix} (\bar{p} - \bar{q}) \equiv sM(\bar{p} - \bar{q}) \tag{12}$$

We shall only consider SAR systems with square pixels in this paper. It is instructive to consider the overall algebraic structure of the SAR imaging transformation. Prior to projection into the slant plane we have

$$f: \Re^3 \to \Re^3: \bar{b} = f(\bar{p}) \text{ by } \bar{b} = sT_{rot}(\bar{p} - \bar{q}) \text{ where } T_{rot} \in SO(\Re,3) \tag{13}$$

This transformation is a special case of an affine 3D to 3D transformation, since the matrix is taken from the special orthogonal group (i.e., othronormal matrices) of dimension 3 over $\Re$, which is a subgroup of the general linear group $GL(\Re,3)$ of 3x3 invertible matrices over $\Re$. This transformation is a group action in 3D space. To get the SAR transformation this group action is followed by a direct subspace projection using the projection matrix

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \tag{14}$$

into the slant plane (or $\Re^2$) as given above. Note that by use of homogenous coordinates[20] we can express the affine transformation of a point $\bar{p}$ via a single homogenous 4x4 transformation matrix T_h as

$$\bar{b}_h = sT_h\bar{p}_h = s \begin{bmatrix} \hat{u}^r & -\hat{u} \cdot \bar{q} \\ \hat{v}^r & -\hat{v} \cdot \bar{q} \\ \hat{n}^r & -\hat{n} \cdot \bar{q} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \bar{p} \\ 1 \end{bmatrix} \tag{15}$$

and the SAR transformation can be expressed as a 2x4 matrix operating on 3D points expressed in homogenous coordinates via

$$f: \Re^3 \to \Re^2: f(\bar{p}) = \bar{\alpha}, \ \bar{\alpha} = s \begin{bmatrix} \hat{u}^r & -\hat{u} \cdot \bar{q} \\ \hat{v}^r & -\hat{v} \cdot \bar{q} \end{bmatrix} \begin{bmatrix} \bar{p} \\ 1 \end{bmatrix} \tag{16}$$

Thus there are 6 degrees of freedom for the SAR transformation needed to specify the location (3 degrees of freedom in $\bar{q}$), orientation (2 degrees of freedom in the rotation matrix T_{rot}), and the coordinate scaling (1 degree of freedom) in the slant plane. Note that the translation component of the SAR transformation matrix will cancel out for line segments

$$\bar{\alpha}_i - \bar{\alpha}_j = s \begin{bmatrix} u_x & u_y & u_z & -\hat{u} \cdot \bar{q} \\ v_x & v_y & v_z & -\hat{v} \cdot \bar{q} \end{bmatrix} \left(\begin{bmatrix} \bar{p}_i \\ 1 \end{bmatrix} - \begin{bmatrix} \bar{p}_j \\ 1 \end{bmatrix} \right) = s \begin{bmatrix} \hat{u}^r \\ \hat{v}^r \end{bmatrix} (\bar{p}_i - \bar{p}_j) = sM(\bar{p}_i - \bar{p}_j) \tag{17}$$

making it convenient to work with distances between points instead of individual points since by doing so we immediately obtain 3D translation invariance. Now that we have derived the essential transformation information, we can explore invariant constraints. We can easily relate the distance between any two distinct points in the SAR image to their 3D distance as

$$d_2(\bar{\alpha}_i, \bar{\alpha}_j) = \|\bar{\alpha}_i - \bar{\alpha}_j\| = \|sM(\bar{p}_i - \bar{p}_j)\| = \|sMd_3(\bar{p}_i, \bar{p}_j)\hat{p}_{ji}\| = d_3(\bar{p}_i, \bar{p}_j)\|sM\hat{p}_{ji}\| \tag{18}$$

where $\hat{p}_{ji}$ is the unit vector in the $\bar{p}_i - \bar{p}_j$ direction, and $d_k(\bullet)$ is Euclidean distance in k dimensions. Therefore a ratio of distances between 2 distinct pairs of SAR image points

$$\frac{d_2(\bar{\alpha}_1, \bar{\alpha}_2)}{d_2(\bar{\alpha}_3, \bar{\alpha}_4)} = \frac{d_3(\bar{p}_1, \bar{p}_2)\|M\hat{p}_{21}\|}{d_3(\bar{p}_3, \bar{p}_4)\|M\hat{p}_{43}\|} \tag{19}$$

will be invariant to the SAR imaging geometry provided

$$f(M, \hat{p}_{21}, \hat{p}_{43}) = \frac{\|M\hat{p}_{21}\|}{\|M\hat{p}_{43}\|} = constant \tag{20}$$

Since we seek 3D point configurations that will satisfy this constraint for arbitrary M, it is obvious that one such condition is that the distinct point pairs form parallel lines in 3 space, i.e., $\hat{p}_{21} = \pm \hat{p}_{43}$. Under this condition we would have an absolute invariant and the constant would be unity in this case. Whether this sufficient condition for an absolute invariant is also a necessary condition remains to be determined.

We shall show that (20) is also a necessary condition for absolute invariance. To prove this, we shall proceed as above for the HRR case. For ease of notation, replace $\hat{p}_{21}$ and $\hat{p}_{43}$ with $\hat{p}$ and $\hat{q}$ respectively. Then we have

$$f(M, \hat{p}, \hat{q}) = \frac{\|M\hat{p}\|}{\|M\hat{q}\|} = \frac{\left\| \begin{bmatrix} \hat{u}^T \\ \hat{v}^T \end{bmatrix} \hat{p} \right\|}{\left\| \begin{bmatrix} \hat{u}^T \\ \hat{v}^T \end{bmatrix} \hat{q} \right\|} = \frac{\sqrt{(\hat{u} \cdot \hat{p})^2 + (\hat{v} \cdot \hat{p})^2}}{\sqrt{(\hat{u} \cdot \hat{q})^2 + (\hat{v} \cdot \hat{q})^2}} \tag{21}$$

and we are interested in bounds on f for arbitrary $\hat{u}$ and $\hat{v}$. Note that both the numerator and denominator of this expression lie between 0 and 1, since each is the magnitude of the projection of a 3D unit vector into a 2D subspace. Now let $\hat{u} = \hat{p}$. Then

$$f = \frac{1}{\sqrt{(\hat{u} \cdot \hat{q})^2 + (\hat{v} \cdot \hat{q})^2}} \geq 1 \tag{22}$$

For $\hat{u} = \hat{q}$ we obtain

$$f = \frac{\sqrt{(\hat{u} \cdot \hat{p})^2 + (\hat{v} \cdot \hat{p})^2}}{1} \leq 1 \tag{23}$$

Therefore for arbitrary $\hat{u}$ and $\hat{v}$, $f = 1$, which in turn implies

$$(\hat{u} \cdot \hat{p})^2 + (\hat{v} \cdot \hat{p})^2 = (\hat{u} \cdot \hat{q})^2 + (\hat{v} \cdot \hat{q})^2 \tag{24}$$

Let $\hat{u} = \hat{p}$. Then

$$1 = (\hat{p} \cdot \hat{q})^2 + (\hat{v} \cdot \hat{q})^2 \tag{25}$$

Let $\hat{u} = \hat{q}$. Then

$$(\hat{q} \cdot \hat{p})^2 + (\hat{v} \cdot \hat{p})^2 = 1 \tag{26}$$

Hence for arbitrary $\hat{u}$

$$(\hat{v} \cdot \hat{q})^2 = (\hat{v} \cdot \hat{p})^2 \Rightarrow (\hat{v} \cdot \hat{q} - \hat{v} \cdot \hat{p})(\hat{v} \cdot \hat{q} + \hat{v} \cdot \hat{p}) = 0 \Rightarrow \hat{v} \cdot \hat{q} = \pm \hat{v} \cdot \hat{p} \tag{27}$$

and since $\hat{v}$ is also arbitrary, we obtain $\hat{p} = \pm \hat{q}$ or parallel vectors. Hence parallel vectors is a necessary and sufficient condition for an absolute ratio distance invariant in SAR imagery.

3.1 Generalizations of the parallel vector invariant

Having established the invariance of parallel vectors under HRR and SAR imaging geometry, a generalization of this result immediately suggests itself. It is clear that two triangular areas (i.e., two triples of co-planar scattering centers) lying in parallel planes also project invariantly (figure 4).
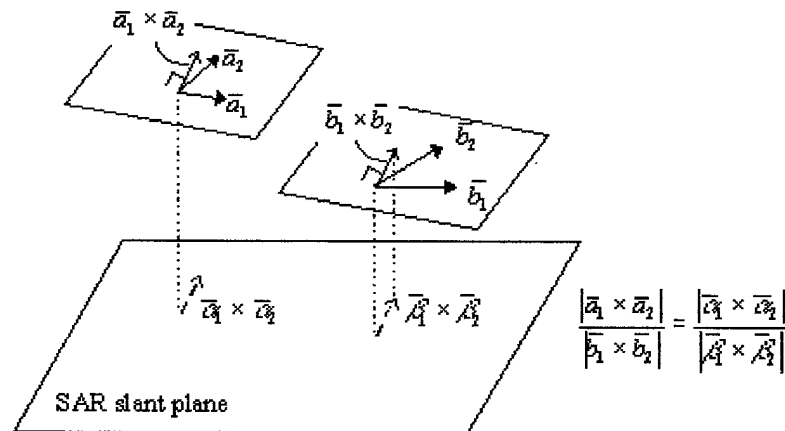

Figure 4. SAR projection of triangular areas lying in parallel planes

This is because the area of a triangle is proportional to the magnitude of the cross product of any two of it's sides expressed as vectors in 3 space (figure 5). Since the cross product is perpendicular to it's base vectors, then any cross product pairs taken from two triangular areas lying in parallel planes will also be parallel, and by the results of the previous sections, the ratio of the magnitudes of these cross product vectors will be preserved under either the SAR or HRR transformation.
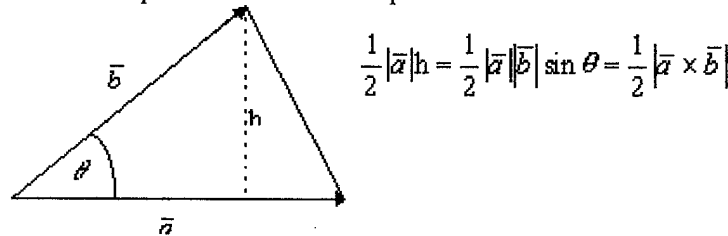

$$\frac{1}{2}|\bar{a}|h = \frac{1}{2}|\bar{a}||\bar{b}|\sin\theta = \frac{1}{2}|\bar{a}\times\bar{b}|$$

Figure 5. Area of triangle formed by vectors $\bar{a}$ and $\bar{b}$

Since the SAR transformation substantially preserves the scattering center geometry, this observation may prove useful as a feature in SAR imagery. Thus there would be a rich set of potential invariants in the case of two parallel planes of coplanar scattering centers, a situation that may be common in SAR imagery of vehicles and other man-made structures. Note that 4 coplanar points with no three colinear would also give rise to invariants as a special case of the above, since there would be 4 distinct triangles of scattering centers. From 4 distinct triangles 6 independent ratio invariants can be formed.

2. SAR quasi-invariants

In many cases it may not be wise necessary or even to demand an absolute invariant to obtain a feature useful for recognition in SAR imagery. This is because the fluctuation in scattering center amplitude and phase center location as a function of SAR imaging geometry is such that scattering peak location persistence is limited to a few degrees in most cases.[21] It is therefore prudent to examine whether a more general geometric configuration can be found that provides feature stability with pose in a local sense. Such local invariants are termed quasi-invariants by Binford.[22]

Following Binford's definition of quasi-invariant, to determine the conditions under which ratios of distances in a SAR image give rise to a quasi-invariant, we expand our expression for a SAR image distance ratio as a multivariate Taylor series in the imaging transformation variables $\hat{u}$ and $\hat{v}$. We then examine conditions on $\hat{p}$ and $\hat{q}$ under which the coefficients of the linear terms vanish in order to determine quasi-invariant ratios. For the purposes of this analysis, it is sufficient to consider f^2 since constant f^2 implies constant f. We select fixed $\hat{u}$ and $\hat{v}$ at $\hat{u}_0$ and $\hat{v}_0$ respectively. Thus

$$f^2 = \frac{(\hat{u}\cdot\hat{p})^2 + (\hat{v}\cdot\hat{p})^2}{(\hat{u}\cdot\hat{q})^2 + (\hat{v}\cdot\hat{q})^2} = f^2\Big|_{\hat{u}_0,\hat{v}_0} + \nabla_u f^2\cdot\hat{u}\Big|_{\hat{u}_0,\hat{v}_0}|\hat{u}-\hat{u}_0| + \nabla_v f^2\cdot\hat{v}\Big|_{\hat{u}_0,\hat{v}_0}|\hat{v}-\hat{v}_0| + O\left(\nabla^2 f^2\right)$$

. (28)

We are interested in determining $\hat{p}$ and $\hat{q}$ that satisfy

$$\nabla_u f^2 \cdot \hat{u}\Big|_{\hat{u}_0, \hat{v}_0} = 0$$

$$\nabla_v f^2 \cdot \hat{v}\Big|_{\hat{u}_0, \hat{v}_0} = 0 \tag{29}$$

that is, finding $\hat{p}$ and $\hat{q}$ that force the coefficient of the linear term in the Taylor expansion to vanish. For ease of notation we drop the 0 subscript on $\hat{u}$ and $\hat{v}$. Note that since f^2 is symmetric with respect to $\hat{u}$ and $\hat{v}$, then ∇f^2 will also be symmetric with respect to $\hat{u}$ and $\hat{v}$. Thus we will get the same constraint result on $\hat{p}$ and $\hat{q}$ from forcing either linear coefficient to vanish. Hence

$$\nabla_u f^2 \cdot \hat{u} = \frac{\left((\hat{u}\cdot\hat{q})^2 + (\hat{v}\cdot\hat{q})^2 \right) 2 (\hat{u}\cdot\hat{p})\hat{p} - \left((\hat{u}\cdot\hat{p})^2 + (\hat{v}\cdot\hat{p})^2 \right) 2 (\hat{u}\cdot\hat{q})\hat{q}}{\left((\hat{u}\cdot\hat{q})^2 + (\hat{v}\cdot\hat{q})^2 \right)^2} \cdot \hat{u}$$

$$= -2 \frac{\left((\hat{u}\cdot\hat{q})^2 + (\hat{v}\cdot\hat{q})^2 \right)(\hat{u}\cdot\hat{p})^2 - \left((\hat{u}\cdot\hat{p})^2 + (\hat{v}\cdot\hat{p})^2 \right)(\hat{u}\cdot\hat{q})^2}{\left((\hat{u}\cdot\hat{q})^2 + (\hat{v}\cdot\hat{q})^2 \right)^2}$$

$$= -2 \frac{(\hat{v}\cdot\hat{q})^2(\hat{u}\cdot\hat{p})^2 - (\hat{v}\cdot\hat{p})^2(\hat{u}\cdot\hat{q})^2}{\left((\hat{u}\cdot\hat{q})^2 + (\hat{v}\cdot\hat{q})^2 \right)^2}$$

$$= -2 \frac{\left((\hat{v}\cdot\hat{q})(\hat{u}\cdot\hat{p}) \right)^2 - \left((\hat{v}\cdot\hat{p})(\hat{u}\cdot\hat{q}) \right)^2}{\left((\hat{u}\cdot\hat{q})^2 + (\hat{v}\cdot\hat{q})^2 \right)^2} = 0 \tag{30}$$

Note that the denominator cannot vanish for non-trivial $\hat{q}$ provided $\hat{q} \neq \pm\hat{n}$. In a similar manner the denominator of $\nabla_v f^2 \cdot \hat{v}$ will not vanish provided $\hat{p} \neq \pm\hat{n}$. Note that the case of $\hat{p}$ and $\hat{q}$ both parallel to $\hat{n}$ would obviously give an invariant for any choice of $\hat{n}$, so we can safely assume a non-trivial denominator for at least one of the linear terms in the Taylor series expansion. Equation (30) then reduces to

$$(\hat{v}\cdot\hat{q}\,\hat{u}\cdot\hat{p} - \hat{v}\cdot\hat{p}\,\hat{u}\cdot\hat{q})(\hat{v}\cdot\hat{q}\,\hat{u}\cdot\hat{p} + \hat{v}\cdot\hat{p}\,\hat{u}\cdot\hat{q}) = 0 \tag{31}$$

and thus

$$\hat{v}\cdot\hat{q}\,\hat{u}\cdot\hat{p} = \pm\hat{v}\cdot\hat{p}\,\hat{u}\cdot\hat{q} \tag{32}$$

If we assume that none of the terms vanish then

$$\frac{\hat{v}\cdot\hat{q}}{\hat{u}\cdot\hat{q}} = \pm\frac{\hat{v}\cdot\hat{p}}{\hat{u}\cdot\hat{p}} \tag{33}$$

Note that each side of this equation can be interpreted as the slope of the projection of a three space vector ($\hat{p}$ or $\hat{q}$) into a 2D subspace (the $(\hat{u}, \hat{v})$ plane). Hence the geometric interpretation of this condition is that the projections of $\hat{p}$ and $\hat{q}$ into the slant plane are either parallel (+), or that they possess mirror symmetry in slope (-), e.g., that have slopes of equal magnitude but opposite sign. We can therefore state that point sets that generate line segment pairs with slopes of equal magnitude when projected into the slant plane are a sufficient condition for a geometric quasi-invariant. Note that the $\hat{n}$ component of $\hat{p}$ and $\hat{q}$ are unconstrained in this case. The strength of this quasi-invariant (that is, the viewing region over which it remains approximately constant) would clearly depend on the relative magnitudes of the $\hat{n}$ components of $\hat{p}$ and $\hat{q}$, as a large $\hat{n}$ component would give the feature more 'wobble' as the imaging geometry is varied in azimuth about the target.

4. CONCLUSIONS AND FUTURE WORK

This paper introduced the concept of pose invariant features for object recognition in SAR imagery, and showed some basic 3D geometric arrangements that give rise to such features. The results show that at least theoretically, parallelism and coplanarity of scattering center locations gives rise to potential SAR image geometric (absolute) invariants. It remains to be seen whether these configurations provide a sufficiently rich set of geometric relationships to allow for a useful number of SAR image invariants for vehicle object recognition. This will be the subject of future research. A potentially fruitful direction is to consider the use of general invariant constraints that arise between affine 3D to 3D transformations and affine 2D to 2D transformations for 3D point sets and their corresponding 2D point set images under the action of the SAR image projection operation. This would be similar to research being done by Weiss for the more general case of the affine and projective optical camera.[23,24,25]

The MSTAR program has recently released a simplified version of a 3D scattering center based SAR synthetic target signature image prediction tool for T72, BMP2, and BTR70 vehicles called <u>Predict Lite</u>. Using the Predict Lite 3D scattering center signature representation of these vehicles, it will be possible to analyze 3D scattering center configurations of a given target object to determine potential SAR image geometric invariants. The utility of such potential invariants can be assessed by examining 2D peaks in Predict Lite synthetic imagery generated from the corresponding 3D scattering center data base. If the synthetic imagery suggests that the invariants have recognition discrimination power, then the invariants can be evaluated using measured SAR imagery of tactical vehicles recently released by <u>DARPA/ISO and AFRL</u>.

As part of the research plan outlined above, it will also be necessary to assess the impact of certain signal processing effects in the SAR image formation process on invariant stability and accuracy. Major SAR image formation effects likely to be important are the radar 2D impulse response, scintillation, discretization, and antenna weighting. These effects are similar to impulse response and radial distortion effects in optical sensors. Experience has shown that accurate calculation of optical image invariants requires compensation for such optics distortions, so it is reasonable to expect that the analogous situation will arise in SAR imagery.[26]

Since the SAR sensor has a finite 2D impulse response, scattering energy that would be concentrated at a point in an ideal infinite resolution sensor is instead spread out over the resulting image in the well known $\sin(x)/x$ pattern. This has obvious practical implications for the stability of image invariants derived from the 3D scattering center representation in Predict Lite. Fortunately Predict Lite includes the SAR impulse response as part of the synthetic image generation process, so that the impact of this phenomenon can be assessed using synthetic data. Antenna weighting functions have an impact similar to the effects of the 2D impulse response.

Scintillation is a phenomena that arises as a consequence of the finite spatial resolution in SAR. A given range and azimuth bin may contain contributions from many small, unresolved scatters which when coherently summed together cause a net response that is highly variable with small changes in the SAR imaging geometry (i.e., the 6 parameters of the SAR imaging transformation from section 3). This will tend to make some 3D scattering centers project in an unreliable manner into the SAR image.

The fact that a SAR image is a discrete representation of an underlying continuous function will effect the accuracy and stability of invariant values. Each image pixel represents a coherent average of the underlying scattering energy with it, and this will clearly tend to shift scattering positions away from their idealized 2D locations as predicted by the basic SAR projective geometry model. Use of state-of-the-art SAR sub-pixel peak detection algorithms may mitigate these effects somewhat, but they cannot eliminate them altogether.

5. ACKNOWLEDGEMENTS

6. REFERENCES

1. L. Novak, G. Benitz, G. Owirka, and L. Bessette, "ATR performance using enhanced resolution SAR," *Algorithms for Synthetic Aperture Radar Imagery III*, E. Zelnio and R. Douglas, eds., v. 2757, paper # 2757-20, pp. 332-337, SPIE Proceedings, Orlando, FL, 1996.

2. G. Owirka and L. Novak, "New SAR ATR algorithm suite," *Algorithms for Synthetic Aperture Radar Imagery*, D. Giglio, ed., v. 2230, pp. 336-343, SPIE Proceedings, Orlando, FL, 1994.

3. T. Binford and T. Levitt, "Model-based recognition of objects in complex scenes," *DARPA Image Understanding Workshop Proceedings*, T. Strat, ed., pp. 89-100, Morgan Kaufmann, Palm Springs, CA, 1996.

4. K. Ikeuchi, M. Wheeler, T. Yamazaki, and T. Shakunaga, "Model Based SAR ATR System," *DARPA Image Understanding Workshop Proceedings*, T. Strat, ed., pp. 1263-1276, Morgan Kauffmann, Palm Springs, CA, 1996.

5. E. Keydel, S. Lee, and J. Moore, "MSTAR extended operating conditions: a tutorial," *Algorithms for Synthetic Aperture Radar Imagery III*, E. Zelnio and R. Douglas, eds., v. 2757, paper #2757-45, pp. 228-242, SPIE Proceedings, 1996.

6. B. Wang and T. Binford, "Generic Model Based Estimation and Detection of Peaks in Image Surfaces," *DARPA Image Understanding Workshop Proceedings*, T. Strat, ed., pp. 913-922, Morgan Kauffmann, Palm Springs, CA, 1996.

7. R. Bhalla, J. Moore, and H. Ling, "A Global Scattering Center Representation of Complex Targets Using the Shooting and Bouncing Ray Technique," IEEE Trans. on Antennas and Propagation, v. 45, n. 12, p. 1850, 1996.

8. *User's Manual for xpatch-t*, Demaco Incorporated, 1 June 1995.

9. *DARPA Image Understanding Workshop Proceedings*, O. Firschein ed., Morgan Kaufmann, Washington, DC, 1993.

10. *DARPA Image Understanding Workshop Proceedings*, O. Firschein ed., Morgan Kaufmann, Monterey, CA, 1994.

11. *DARPA Image Understanding Workshop Proceedings*, T. Strat ed., Morgan Kauffmann, Palm Springs, CA, 1996.

12. *DARPA Image Understanding Workshop Proceedings*, T. Strat ed., Morgan Kauffmann, New Orleans, LA, 1997.

13. J. Mundy and A. Zisserman, eds., *Geometric Invariance in Computer Vision*, MIT Press, Cambridge MA, 1992.

14. I. Weiss, "Projective Invariants of Shapes," *DARPA Image Understanding Workshop Proceedings*, pp. 1125-1134, 1988.

15. J. Mundy, A. Zisserman, and D. Forsyth, eds., *Applications of Invariance in Computer Vision*, Springer-Verlag, New York NY, 1994.

16. P. Payton, E. Barrett, W. Kober, J. Thomas, and S. Johnson, "HRR Invariants in Space," *Proceedings Of the Sixth Automatic Target Recognition Systems and Technology Conference*, Hunstville, AL, 1997.

17. T. Binford, "Context and Quasi-Invariants with SAR Imagery," DARPA Image Understanding Program Image Exploitation and ATR focus area kickoff meeting, Dayton, OH, February 26-27, 1997.

18. M. Stuff, "Motion Measuring SAR for moving ground targets," DARPA Image Understanding Program Image Exploitation and ATR focus area technical review meeting, Dayton, OH, September 29-30, 1997.

19. F. Leberl, *Radargrammetric Image Processing*, ch. 4, Artech House, Norwood, MA, 1990.

20. J. Mundy and A. Zisserman, eds., *Geometric Invariance in Computer Vision*, p. 479, MIT Press, Cambridge MA, 1992.

21. G. Jones and B. Bhanu, "Invariants for the recognition of articulated and occluded objects in SAR images," *DARPA Image Understanding Workshop Proceedings*, T. Strat, ed., v. II, pp. 1135-1144, Morgan Kaufmann, New Orleans, LA, 1997.

22. T. Binford and T. Levitt, "Quasi-Invariants: Theory and Exploitation," *DARPA Image Understanding Workshop Proceedings*, O. Firschein, ed., pp. 819-830, Morgan Kaufmann, Washington, DC, 1993.

23. I. Weiss, "Model-Based Recognition of 3D Curves from One View," *DARPA Image Understanding Workshop Proceedings*, T. Strat, ed., pp. 1251-1256, Morgan Kauffmann, Palm Springs, CA, 1996.

24. I. Weiss, "3D Object Recognition from Multiple and Single Views," *DARPA Image Understanding Workshop Proceedings*, T. Strat, ed., v. II, pp. 1041-1046, Morgan Kaufmann, New Orleans, LA, 1997.

25. I. Weiss, "Model-Based ATR from Single and Multiple Views," DARPA Image Understanding Program Image Exploitation and ATR focus area technical review meeting, Dayton, OH, September 29-30, 1997.

26. J. Mundy and A. Zisserman, eds., *Geometric Invariance in Computer Vision*, p. 511, MIT Press, Cambridge MA, 1992.